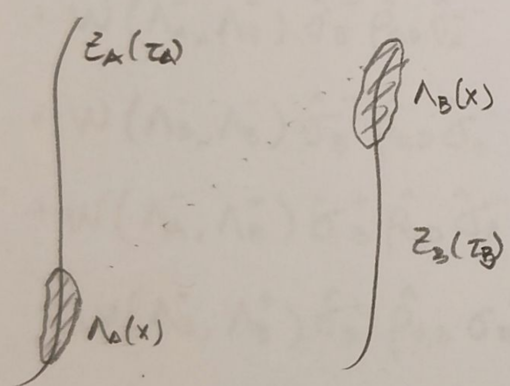
Quantum Information Protocols in QFT

- Our goal: couple local probes to a QFT to implement QI protocols.

Quantum Information Protocols

- Rely on 2 (or more) detectors.

Let us consider 2 particle detectors



$$\hat{H}_A = \Omega_A \hat{\sigma}_A^+ \hat{\sigma}_A^- = i\frac{d}{d\tau_A}$$

$$\hat{H}_B = \Omega_B \hat{\sigma}_B^+ \hat{\sigma}_B^- = i\frac{d}{d\tau_B}$$

are the free Hamiltonians generating time evolution w.r.t. τ_A and τ_B. $\to$ FNC.

The interactions with the field are prescribed by the interaction Hamiltonian density:

$$\hat{h}_I(x) = \lambda \Lambda_A(x) \hat{\mu}_A(\tau_A) \hat{\phi}(x) + \lambda \Lambda_B(x) \hat{\mu}_B(\tau_B) \hat{\phi}(x)$$

$\hookrightarrow e^{i\Omega_A\tau_A}\hat{\sigma}_A^+ + e^{-i\Omega_A\tau_A}\hat{\sigma}_A^-$ $\quad\hookrightarrow e^{i\Omega_B\tau_B}\hat{\sigma}_B^+ + e^{-i\Omega_B\tau_B}\hat{\sigma}_B^-$.

Same as before: $\hat{\rho}_0 = \underbrace{\hat{\rho}_{0,A} \otimes \hat{\rho}_{0,B}}_{\hat{\rho}_{0,D}} \otimes \hat{\rho}_w \longrightarrow$ quasifree.

$$\hat{U}_I = T\exp\left(-i\int dV\, \hat{h}_I(x)\right),$$

$$\hat{\rho}_D = \mathrm{tr}_\phi\left(\hat{U}_I \rho_0 \hat{U}_I^\dagger\right), \quad \hat{U}_I = \mathbb{1} + U_I^{(1)} + U_I^{(2)} + \ldots$$

$$\mathrm{tr}_\phi\left(U_I^{(1)}\hat{\rho}_0 U_I^{(1)\dagger}\right) = \lambda^2\int dV\,dV'\left(\Lambda_A(x)\hat{\mu}_A(\tau_A)+\Lambda_B(x)\hat{\mu}_B(\tau_B)\right)\hat{\rho}_{0,D}\cdot$$

$$\times\left(\Lambda_A(x')\hat{\mu}_A(\tau_A')+\Lambda_B(x')\hat{\mu}_B(\tau_B')\right)$$

$$\times\ \omega\left(\hat{\phi}(x)\,\hat{\rho}_\omega\,\hat{\phi}(x')\right) \to W(x',x)$$

$$= \lambda^2\Big(W(\Lambda_A^-,\Lambda_A^-)\hat{\sigma}_A^-\hat{\rho}_{0,D}\hat{\sigma}_A^- + W(\Lambda_A^+,\Lambda_A^-)\hat{\sigma}_A^-\hat{\rho}_{0,D}\sigma_A^+$$

$$+ W(\Lambda_B^-,\Lambda_A^-)\hat{\sigma}_A^-\hat{\rho}_{0,D}\hat{\sigma}_B^- + W(\Lambda_B^+,\Lambda_A^-)\hat{\sigma}_A^-\hat{\rho}_{0,D}\sigma_B^+$$

$$+ W(\Lambda_A^-,\Lambda_A^+)\hat{\sigma}_A^+\hat{\rho}_{0,D}\hat{\sigma}_A^- + W(\Lambda_A^+,\Lambda_A^+)\hat{\sigma}_A^+\hat{\rho}_{0,D}\hat{\sigma}_A^+$$

$$+ W(\Lambda_B^-,\Lambda_A^+)\hat{\sigma}_A^+\hat{\rho}_{0,D}\hat{\sigma}_B^- + W(\Lambda_B^+,\Lambda_A^+)\hat{\sigma}_A^+\hat{\rho}_{0,D}\hat{\sigma}_B^+$$

$$+ W(\Lambda_A^-,\Lambda_B^-)\hat{\sigma}_B^-\hat{\rho}_{0,D}\hat{\sigma}_A^- + W(\Lambda_A^+,\Lambda_B^-)\hat{\sigma}_B^-\hat{\rho}_{0,D}\hat{\sigma}_A^+$$

$$+ W(\Lambda_B^-,\Lambda_B^-)\hat{\sigma}_B^-\hat{\rho}_{0,D}\hat{\sigma}_B^- + W(\Lambda_B^+,\Lambda_B^-)\hat{\sigma}_B^-\hat{\rho}_{0,D}\hat{\sigma}_B^+$$

$$+ W(\Lambda_A^-,\Lambda_B^+)\hat{\sigma}_B^+\hat{\rho}_{0,D}\hat{\sigma}_A^- + W(\Lambda_A^+,\Lambda_B^+)\hat{\sigma}_B^+\hat{\rho}_{0,D}\hat{\sigma}_A^+$$

$$+ W(\Lambda_B^-,\Lambda_B^+)\hat{\sigma}_B^+\hat{\rho}_{0,D}\sigma_B^- + W(\Lambda_B^+,\Lambda_B^+)\hat{\sigma}_B^+\hat{\rho}_{0,D}\hat{\sigma}_B^+\Big)$$

$$= \lambda^2\sum_{\substack{\lambda_1,\lambda_2=\pm\\ I,J=A,B}} W(\Lambda_I^{\lambda_1},\Lambda_J^{\lambda_2})\,\hat{\sigma}_J^{\lambda_2}\hat{\rho}_{0,D}\hat{\sigma}_I^{\lambda_1}. \qquad \Lambda_J^\pm(x) = e^{\pm i\Omega_I\tau_I}\Lambda_I(x)$$

$$\mathrm{tr}_\phi\left(U_I^{(2)}\hat{\rho}_0\right) = -\lambda^2\int dV\,dV'\,\theta(\tau-\tau')\left(\Lambda_A(x)\hat{\mu}_A(\tau_A)+\Lambda_B(x)\hat{\mu}_B(\tau_B)\right)$$

$$\times\left(\Lambda_A(x')\hat{\mu}_A(\tau_A')+\Lambda_B(x')\hat{\mu}_B(\tau_B')\right)\rho_{0,D}$$

$$\times\ \mathrm{tr}_\phi\left(\phi(x)\phi(x')\hat{\rho}_\omega\right)$$

$$= -\lambda^2\sum_{\substack{\lambda_1,\lambda_2=\pm\\ I,J=A,B}} W_\tau(\Lambda_I^{\lambda_1},\Lambda_J^{\lambda_2})\,\hat{\sigma}_I^{\lambda_1}\hat{\sigma}_J^{\lambda_2}\hat{\rho}_{0,D}\ , \qquad \sigma_I^+\sigma_I^+ = \sigma_I^-\sigma_I^- = 0$$

$$\mathrm{tr}_\phi\left(\hat{\rho}_0 U_I^{(2)\dagger}\right) = -\lambda^2\sum_{\substack{\lambda_1,\lambda_2=\pm\\ I,J=A,B}} \left(W_\tau(\Lambda_J^{\lambda_1},\Lambda_J^{\lambda_2})\right)^*\hat{\rho}_{0,D}\hat{\sigma}_J^{\lambda_2}\hat{\sigma}_I^{\lambda_1}$$

$$\hat\rho_D = \hat\rho_{0,D} + \lambda^2 \sum_{\substack{\Delta_1,\Delta_2=\pm \\ I,J=A,B}} \Big(W(\Lambda_I^{\Delta_1}, \Lambda_J^{\Delta_2})\, \hat\sigma_J^{\Delta_2} \hat\rho_{0,D} \hat\sigma_I^{\Delta_1}$$

$$- W_t(\Lambda_I^{\Delta_1}, \Lambda_J^{\Delta_2})\, \hat\sigma_I^{\Delta_1} \hat\sigma_J^{\Delta_2} \hat\rho_{0,D}$$

$$- \big(W_t(\Lambda_I^{\Delta_1}, \Lambda_J^{\Delta_2})\big)^* \hat\rho_{0,D} \hat\sigma_J^{\Delta_2} \hat\sigma_I^{\Delta_1} \Big) + \mathcal{O}(\lambda^4),$$

which is most definitely a quantum channel.

<u>ex</u>: $\hat\rho_{A,0} = \hat\sigma_A^+ \hat\sigma_A^- = |e_A\rangle\langle e_A|$, $\hat\rho_{B,0} = \hat\sigma_B^- \hat\sigma_B^+ = |g_B\rangle\langle g_B|$

$$\Rightarrow \hat\rho_D = \hat\sigma_A^+ \hat\sigma_A^- \hat\sigma_B^- \hat\sigma_B^+ + \lambda^2 \Big(\hat\sigma_A^- \hat\sigma_A^+ \hat\sigma_B^- \hat\sigma_B^+ \, W(\Lambda_A^+, \Lambda_A^-)$$

$$+ \hat\sigma_A^+ \hat\sigma_A^- \hat\sigma_B^+ \hat\sigma_B^- \, W(\Lambda_B^-, \Lambda_B^+)$$

$$+ \hat\sigma_A^+ \hat\sigma_B^+ \, W(\Lambda_A^+, \Lambda_B^+)$$

$$+ \hat\sigma_A^- \hat\sigma_B^- \, W(\Lambda_B^-, \Lambda_A^-)$$

$$- \hat\sigma_A^+ \hat\sigma_A^- \hat\sigma_B^- \hat\sigma_B^+ \, W_t(\Lambda_A^+, \Lambda_A^-) - \text{H.c.}$$

$$- \sigma_A^+ \sigma_A^- \hat\sigma_B^- \hat\sigma_B^+ \, W_t(\Lambda_B^-, \Lambda_B^+) - \text{H.c.}$$

$$- \hat\sigma_A^- \hat\sigma_B^+ \, W_t(\Lambda_A^-, \Lambda_B^+) - \text{H.c.}$$

$$- \hat\sigma_A^- \hat\sigma_B^+ \, W_t(\Lambda_B^+, \Lambda_A^-) - \text{H.c.} \Big) + \mathcal{O}(\lambda^4)$$

$$= \hat\rho_{0,D} + \lambda^2 \Big(\hat\sigma_A^- \hat\sigma_A^+ \hat\sigma_B^- \hat\sigma_B^+ W_{AA}^{+-} + \hat\sigma_A^+ \hat\sigma_A^- \hat\sigma_B^+ \hat\sigma_B^- W_{BB}^{-+}$$

$$+ \hat\sigma_A^+ \hat\sigma_B^+ W_{AB}^{++} + \hat\sigma_A^- \hat\sigma_B^- W_{BA}^{--}$$

$$- \hat\sigma_A^+ \hat\sigma_A^- \hat\sigma_B^- \hat\sigma_B^+ \big(W_{AA}^{+-} + W_{BB}^{-+} \big)$$

$$- \hat\sigma_A^- \hat\sigma_B^+ G_{AB}^{-+} - \hat\sigma_A^+ \hat\sigma_B^- \big(G_{AB}^{-+}\big)^* \Big) + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} \lambda^2 W_{AA}^{+-} & & & \lambda^2 W_{AB}^{++} \\ & & (G_{AB}^{-+})^* & \\ & G_{AB}^{-+} & 1-\lambda^2(W_{AA}^{+-} + W_{BB}^{-+}) & \\ \lambda^2 W_{BA}^{--} & & & \lambda^2 W_{BB}^{-+} \end{pmatrix} + \mathcal{O}(\lambda^4)$$

A deexcitation prob.

non-local correlations in the antidiagonal.

B excitation prob.

The excitation probability of B is independent of A!

In fact, $\hat{\rho}_B = \mathrm{tr}_A(\hat{\rho}_D) = \begin{pmatrix} 1-\lambda^2 W_{BB}^{-+} & \\ & \lambda^2 W_{BB}^{-+} \end{pmatrix} + \mathcal{O}(\lambda^4)$.

↳ this is because A's effect on the state of $\hat{\phi}$ is of order λ^2 $\Rightarrow$ B feels $W + \lambda^2 \delta W_A$, so $\lambda^2 W$ is unchanged to leading order.

→ Both Alice and Bob are in states such that

$$\langle \hat{h}_{I,A}(x) \rangle = 0 \;\&\; \langle \hat{h}_{I,B}(x) \rangle = 0.$$

In general, if the initial states for A and B are $|\psi_A\rangle = \alpha_A |e_A\rangle + \beta_A |g_A\rangle$, $|\psi_B\rangle = \alpha_B |e_B\rangle + \beta_B |g_B\rangle$,

it can be shown that (arXiv:1405.3988)

$$P_B = |\alpha_B|^2 + R_B + S_{AB} + \mathcal{O}(\lambda^4)$$

where

$$S_{AB} = 4\lambda^2 \int dV dV' \Lambda_A(x) \Lambda_B(x') \, \mathrm{Re}(\alpha_A^* \beta_A e^{i\Omega_A \tau_A}) \times \mathrm{Re}(\alpha_B^* \beta_B e^{i\Omega_B \tau_B'} \, i E(x, x'))$$

and $R_B = |\beta_B|^2 P_B(\Omega_B) - |\alpha_B|^2 P_B(-\Omega_B)$.

↳ $P_B(\omega) = \lambda^2 \int dV dV' \Lambda_B(x) \Lambda_B(x') e^{-i\omega(\tau_B - \tau_B')} W(x, x')$

So A can indeed affect the excitation probability of B IF THEY ARE CAUSALLY CONNECTED!.

→ This allows for Alice to send classical information to Bob by either coupling to the field or not.

↳ This is a very noisy protocol!

Classical Channel capacity is:

$$C_{AB} = \lambda^4 \frac{2}{\ln 2} \left(\frac{S_2}{4|\alpha_B||\beta_B|} \right)^2 + \mathcal{O}(\lambda^6)$$

One can also consider instead $\Omega = 0$ for non-perturbative calculations:

$$h_I(x) = \lambda \Lambda(x)\, \hat{\mu}\, \hat{\Phi}(x)$$

for each of the detectors. In this case $\hat{\mu}$ does not depend on τ, and this is very similar to the gapless detector case:

$$T\exp\left(-i\int dV h_I(x)\right) = e^{\sum_{n=1}^{\infty} \hat{\Theta}_n},$$

$$\hat{\Theta}_1 = -i\int dV\, \hat{h}_I(x) = -i\lambda \hat{\mu}_A \hat{\Phi}(\Lambda_A) - i\lambda \hat{\mu}_B \hat{\Phi}(\Lambda_B)$$

$$\hat{\Theta}_2 = \frac{1}{2}\int dV dV' \theta(\tau-\tau')\,[\hat{h}_I(x), \hat{h}_I(x')]$$

$$= -\frac{\lambda^2}{2}\int dV dV' \theta(\tau-\tau')\Big(\hat{\mu}_A^2 \Lambda_A(x)\Lambda_A(x')[\hat{\Phi}(x), \hat{\Phi}(x')] + \hat{\mu}_A\hat{\mu}_B \Lambda_A(x)\Lambda_B(x')[\hat{\Phi}(x), \hat{\Phi}(x')] + \hat{\mu}_A\hat{\mu}_B \Lambda_B(x)\Lambda_A(x')[\hat{\Phi}(x), \hat{\Phi}(x')] + \hat{\mu}_B^2 \Lambda_B(x)\Lambda_B(x')[\hat{\Phi}(x), \hat{\Phi}(x')]\Big)$$

$$= -i\frac{\lambda^2}{2}\Big(G_R(\Lambda_A,\Lambda_A) + G_R(\Lambda_B,\Lambda_B) + \hat{\mu}_A\hat{\mu}_B\big(G_R(\Lambda_A,\Lambda_B) + G_R(\Lambda_B,\Lambda_A)\big)\Big).$$

Also notice that $[\hat{h}_I(x), h_I(x')]$ commutes with $\hat{h}_I(x'')$, so $\hat{\Theta}_3 = 0$.

$$\Rightarrow \hat{U}_I = e^{-i\lambda\hat{\mu}_A\hat{\Phi}(\Lambda_A) - i\lambda\hat{\mu}_B\hat{\Phi}(\Lambda_B) - \underbrace{\frac{i\lambda^2}{2}\Delta_{AA} - \frac{i\lambda^2}{2}\Delta_{BB}}_{\text{these do not matter}} - \frac{i\lambda^2}{2}\Delta_{AB}\hat{\mu}_A\hat{\mu}_B}, \text{ with}$$

$$\Delta_{IJ} = \big(G_R(\Lambda_I,\Lambda_J) + G_R(\Lambda_J,\Lambda_I)\big) = G_R(\Lambda_I,\Lambda_J) + G_A(\Lambda_I,\Lambda_J)$$

$$\Rightarrow \hat{U}_I = e^{-\frac{i\lambda^2}{2}\Delta_{AB}\hat{\mu}_A\hat{\mu}_B}\, e^{-i\lambda\hat{\mu}_A\hat{\Phi}(\Lambda_A) - i\lambda\hat{\mu}_B\hat{\Phi}(\Lambda_B)}$$