Entanglement Harvesting

- The vacuum of a QFT has correlations between any two regions of spacetime.
  ↳ some* of these actually contain entanglement.
- idea: extract entanglement from the vacuum
  ↳ perturbatively.* → we will see why later
- Two detectors in their ground state gives:

$$\hat{\rho}_D = \begin{pmatrix} 1-\mathcal{L}_{AA}-\mathcal{L}_{BB} & & & \mathcal{M}^* \\ & \mathcal{L}_{BB} & \mathcal{L}_{AB}^* & \\ & \mathcal{L}_{AB} & \mathcal{L}_{AA} & \\ \mathcal{M} & & & \cdot \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\mathcal{L}_{IJ} = \lambda^2 W_{\bar{I}\bar{J}}^{-+}$, $\mathcal{M} = -\lambda^2 G_{IJ}^{++}$. → note that in the video this was incorrect ($\mathcal{M} = -\lambda^2 G_{IJ}^{--}$).

Can this state be entangled?

$$\hat{\rho}_D^{t_A} = \begin{pmatrix} 1-\mathcal{L}_{AA}-\mathcal{L}_{BB} & & & \mathcal{L}_{AB} \\ & \mathcal{L}_{BB} & \mathcal{M} & \\ & \mathcal{M}^* & \mathcal{L}_{AA} & \\ \mathcal{L}_{AB}^* & & & \cdot \end{pmatrix} + \mathcal{O}(\lambda^4)$$

→ eigenvalues:

$$\frac{1}{2}\left(1-\mathcal{L}_{AA}-\mathcal{L}_{BB} \pm \sqrt{4|\mathcal{L}_{AB}|^2+(1-\mathcal{L}_{AA}-\mathcal{L}_{BB})^2}\right)$$ → can only be negative to 4th order

$$\frac{1}{2}\left(\mathcal{L}_{AA}+\mathcal{L}_{BB} \pm \sqrt{4|\mathcal{M}|^2+(\mathcal{L}_{AA}-\mathcal{L}_{BB})^2}\right)$$ → can be negative to <u>2nd</u> order.

$$\Rightarrow \mathcal{N}(\hat{\rho}_D) = \max\left(0, \sqrt{|\mathcal{M}|^2 - \left(\frac{\mathcal{L}_{AA} - \mathcal{L}_{BB}}{2}\right)^2} - \frac{\mathcal{L}_{AA} + \mathcal{L}_{BB}}{2}\right)$$

↳ faithful entanglement quantifyer for 2 qubits.

→ entanglement is a competition between the non-local $\mathcal{M}$ term and the local noise terms $\mathcal{L}_{AA} + \mathcal{L}_{BB}$ (vacuum excitation prob.)

If $\mathcal{L}_{AA} = \mathcal{L}_{BB} = \mathcal{L}$, we then find

$$\mathcal{N}(\hat{\rho}_D) = \max(0, |\mathcal{M}| - \mathcal{L})$$

However, there are two ways in which the detectors can become entangled:

- communication.
- entanglement harvesting.

$$\mathcal{M} = \int dV\, dV'\, \Lambda_A(x)\, \Lambda_B(x')\, e^{i\Omega(\tau_A + \tau_B')} \Big(\underbrace{\tfrac{1}{2} H(x,x')}_{\text{vacuum correlations.}} + \underbrace{\tfrac{i}{2}\Delta(x,x')}_{\substack{G_R + G_A \\ \text{communication}}}\Big)$$

$$= \tfrac{1}{2} H(\Lambda_A^+, \Lambda_B^+) + \tfrac{i}{2}\Delta(\Lambda_A^+, \Lambda_B^-)$$

↓ this is entanglement harvested from the field

↳ this term is due to communication between the detectors through the field

arXiv: 2109.11561 and arXiv: 2212.13262.

One way of considering interactions such that $\Delta(\Lambda_A^+, \Lambda_B^+) = 0$ is to consider the supports of Λ_A and Λ_B to be spacelike* separated. In this case, the $\mathcal{M}$ term will be entirely due to the vacuum correlations.

* → you can also consider approximate space-like separation, so that $\Delta(\Lambda_A^+, \Lambda_B^+) \ll |\mathcal{M}| - \mathcal{L}$, which has the same effect.

→ Show examples!

The no-go theorems of entanglement Harvesting

↳ Eduardo Martín-Martínez and collaborators arXiv:1703.02982, arXiv:1803.11214.

→ Entanglement Breaking Channel

A channel Φ is said to be entanglement breaking if $(\mathbb{1} \otimes \Phi)(\Gamma)$ is separable for every Γ. Φ is entanglement breaking if and only if

$$\Phi(\hat{\rho}) = \sum_k |\psi_k\rangle\langle\psi_k| \langle\phi_k|\hat{\rho}|\phi_k\rangle.$$

→ Simply generated unitaries: $\hat{U} = e^{-i\hat{\mu}\otimes\hat{X}}$

$\hat{\mu}$ observable in system A and $\hat{X}$ an observable in system S.

$$\Rightarrow \hat{\rho}_A = \mathrm{tr}_S(\hat{U}(\hat{\rho}_{0,A} \otimes \hat{\rho}_{0,S})\hat{U}^\dagger)$$

$$= \mathrm{tr}_S\left(e^{-i\hat{\mu}\hat{X}}\, \hat{\rho}_{0,A} \otimes \hat{\rho}_{0,S}\, e^{i\hat{\mu}\hat{X}}\right) \qquad \hat{X}|x_k\rangle = x_k|x_k\rangle$$

$$= \sum_k \langle x_k| e^{-i\hat{\mu}x_k}\, \hat{\rho}_{A,0} \otimes \hat{\rho}_{0,S}\, e^{i\hat{\mu}x_k} |x_k\rangle$$

$$= \sum_k \langle x_k| \hat{\rho}_{0,S} |x_k\rangle\, e^{-i\hat{\mu}x_k}\, \hat{\rho}_{A,0}\, e^{i\hat{\mu}x_k}$$

$\rightarrow$ generalizes for $\hat{X}$ with continuous spectrum.

$\Rightarrow$ δ-coupled and <u>some</u> gapless detectors cannot entangle.

↳ for 2 detectors (gapless), we have:

$$\hat{U} = \mathcal{T} e^{-i\int dV (\hat{\mu}_A \Lambda_A(x) + \hat{\mu}_B \Lambda_B(x))\hat{\Phi}(x)}$$

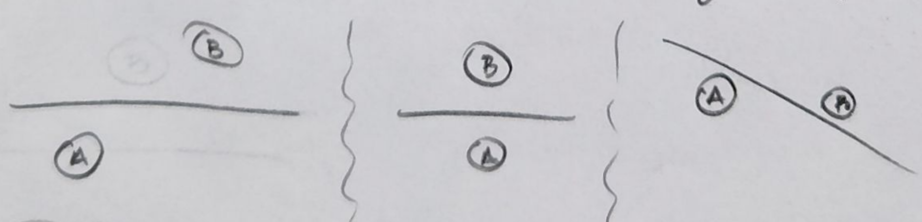
If $\hat{U} = e^{-i\hat{\mu}_A \hat{\Phi}(\Lambda_A)} e^{-i\hat{\mu}_B \hat{\Phi}(\Lambda_B)}$ or $\hat{U} = e^{-i\hat{\mu}_B \hat{\Phi}(\Lambda_B)} e^{-i\hat{\mu}_A \hat{\Phi}(\Lambda_A)}$

the unitary is an entanglement breaking channel in each detector, thus, they cannot become entangled. $\rightarrow$ also applies to δ-coupled.

$\rightarrow$ when does $\hat{U}$ factor as a product?

↳ whenever one can fit a Cauchy surface between the interaction regions!

e.g.:



no entanglement

e.g.:

large interaction times.

entanglement(?) entanglement(?)

$\Rightarrow$ A and B never become entangled if they use δ-couplings. $\rightarrow$ Can always find a Cauchy slice between non-overlapping interactions.

When the interactions between detectors are entanglement breaking, it is possible to compute the "classical" channel capacity for the interactions. If A interacts with the field before B, then

$$C(\mathcal{E}) = H\left(\frac{1}{2} + \frac{e^{-2\lambda^2 W(\Lambda_B,\Lambda_B)}}{2}\cos(2\lambda^2\Delta(\Lambda_A,\Lambda_B))\right) - H\left(\frac{1}{2} - \frac{e^{-2\lambda^2 W(\Lambda_B,\Lambda_B)}}{2}\right),$$

where $H(x) = -x\log_2(x) - (1-x)\log_2(1-x)$.

(and in these cases, of course, $Q(\mathcal{E}) = 0$).