Considerations about Particle Detectors

Main references:

arXiv: 2206.01225 → definition of particle detectors from non-rel. q. systems.
arXiv: 2006.12514 → broken covariance
arXiv: 2102.03408 → faster than light signalling (and arXiv: 2305.07756)

The models we have discussed so far are effective descriptions for localized quantum systems. These descriptions are intrinsically non-relativistic*.
↳ problems with causality & covariance!
↳ these problems are negligible in most cases!

How to describe a non-relativistic quantum system in spacetime?

Consider a quantum system with one position degree of freedom in 3D: $(\hat{\vec{x}}, \hat{\vec{p}})$, $\hat{H}(\hat{x}, \hat{p}, t)$.

In principle, this system is defined in a Galilean spacetime. We need to define it in Minkowski spacetime first.

In an inertial frame which is comoving with the rest frame description of $\hat{H}(\hat{x}, \hat{p}, t)$, not much should change, provided that the non-relativistic energy of the particle is sufficiently smaller than its rest mass:

$$\langle \hat{H}_{NR} \rangle \ll mc^2, \quad \hat{H} = \underbrace{mc^2}_{\text{rest mass}} + \hat{H}_{NR} \longrightarrow \frac{\hat{\vec{p}}^2}{2m} + V(\hat{\vec{x}})$$

↳ very few things change

↳ $\hat{\vec{x}}$ becomes associated with the position degree of freedom in this specific slice.

↳ this description only works with respect to this foliation, comoving with the potential that localizes the particle.

Interaction with a quantum field:

- Let $\hat{\phi}(x) = \hat{\phi}(t, \vec{x})$ be a scalar quantum field
- Let $\mu(\hat{\vec{x}})$ be a position-operator-dependent observable in the LNRQS.

$$H_I(t, \vec{x}) = \lambda \chi(t) \mu(\hat{\vec{x}}) \hat{\phi}(t, \hat{\vec{x}})$$ is the interaction Hamiltonian.

$$= \lambda \chi(t) \int d^3x \, |\vec{x}_t\rangle\langle\vec{x}_t| \, \mu(\hat{\vec{x}}(t)) \, \hat{\phi}(t, \hat{\vec{x}}(t)) \quad , \quad \langle\vec{x}_t| \hat{\vec{x}}(t) |\vec{x}'_t\rangle = \vec{x} \, \delta^{(3)}(\vec{x} - \vec{x}')$$

verify this!

$$|x_t\rangle = e^{i\hat{H}t} |\vec{x}\rangle$$

$$= \lambda \chi(t) \int d^3x \, \mu(\vec{x}) \hat{\phi}(t, \vec{x}) |\vec{x}_t\rangle\langle\vec{x}_t|$$

$$\langle\psi_n|x_t\rangle = e^{iE_nt} \psi_n^*(\vec{x})$$

$$= \sum_{nm} \lambda \chi(t) \int d^3x \, \mu(\vec{x}) \hat{\phi}(t, \vec{x}) |\psi_n\rangle\langle\psi_n|\vec{x}_t\rangle\langle\vec{x}_t|\psi_m\rangle\langle\psi_m|$$

$$= \int d^3x \left(\sum_{nm} \lambda \chi(t) \overbrace{\mu(\vec{x}) \psi_n^*(\vec{x}) \psi_m(\vec{x})}^{f_{nm}(\vec{x})} e^{i\underbrace{(E_n - E_m)}_{\Omega_{nm}}t} \hat{\phi}(t, \vec{x}) |\psi_m\rangle\langle\psi_n| \right)$$

Focusing on two energy levels, we then obtain

$n, m = g, e \, , \; E_g < E_e , \; \Omega \equiv E_e - E_g$

$$\hat{H}_I(t) = \int d^3x \, \lambda \chi(t) \Big(f_{gg}(\vec{x}) |g\rangle\langle g| + f_{ge}(\vec{x}) e^{i\Omega t} |e\rangle\langle g| + f_{eg}(\vec{x}) e^{-i\Omega t} |g\rangle\langle e| + f_{ee}(\vec{x}) |e\rangle\langle e| \Big)$$

The $|g\rangle\langle g|$ and $|e\rangle\langle e|$ terms are basically associated to a gapless detector, and cannot promote energy level transitions.

↳ nothing fun happens to the eigenstates.

If one is mostly interested in transitions between the levels g and e, this becomes exactly a UDW detector.

- one can consider more general fields and quantum systems, being able to reproduce, for example:

  ↳ atoms interacting with E.-M. arXiv:1605.07180
  ↳ nucleons int. with neutrinos, arXiv: 2009.10165, 2111.12779
  ↳ any system int. with lin. q. gravity arXiv: 2106.15641, arXiv: 2210.14921
  ↳ supercond. qubits and E.-M. arXiv: 1709.09684.

What about the $\hat{\vec{x}}$ and $\hat{\vec{p}}$ operators in curved spacetimes?

→ Fermi Normal Coordinates: $z(\tau) \mapsto (\tau, \vec{x})$, Σ_τ

→ $|\Psi(\tau)\rangle \in L^2(\Sigma_\tau)$, $\langle \Psi | \phi \rangle \equiv \int_{\Sigma_\tau} d\Sigma \, \Psi^*(\vec{x}) \phi(\vec{x})$, $d\Sigma = \sqrt{g_\Sigma} d^3x$

→ Build extended Fermi frame e_μ → p. transport over Σ_τ

→ $\hat{x}^i \Psi(\vec{x}) = x^i \Psi(\vec{x})$, $\hat{\vec{x}} = \hat{x}^i e_i$

→ $\hat{p}_i \Psi(\vec{x}) = \dfrac{-i}{(g_\Sigma)^{1/4}} \dfrac{\partial}{\partial x^i} \left((g_\Sigma)^{1/4} \Psi(\vec{x}) \right)$

With these choices, one can show that:

1) $\hat{x}^{i\dagger} = \hat{x}^i$,

2) $\hat{p}_i^\dagger = \hat{p}_i$, provided that $\psi|_{\partial\Sigma_\tau} = 0 \rightarrow \operatorname{supp}\psi \subseteq \Sigma_\tau$.

3) $[\hat{x}^i, \hat{p}_j] = i\,\delta^i_j$

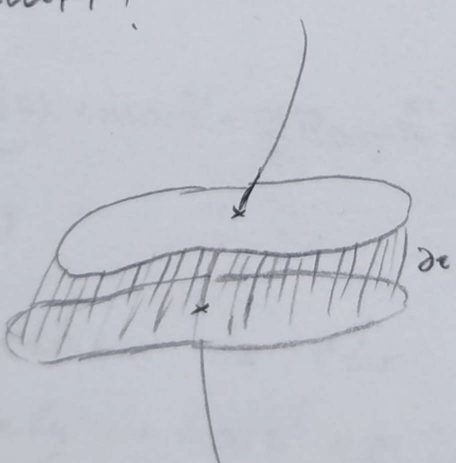
↳ check these

This means that there is a direct translation of the kinematics to curved spacetimes!

What about the dynamics?

If in flat spacetime around an inertial trajectory, one has the Hamiltonian $\hat{H}(\hat{x}, \hat{p}, t)$, is it enough to use the Hamiltonian $\hat{H}(\hat{x}, \hat{p}, \tau)$ with the new $\hat{x}$ and $\hat{\vec{p}}$?

NO! → redshift!



∂_τ is not normal to the surface for $\vec{x} \neq \vec{0}$.

The redshift factor will then be

$$\gamma(x) = \frac{1}{\|d\tau\|} = |g_{\tau\tau} - g_{\tau i} g_{\tau j} h^{ij}|^{1/2}$$

Classically, one would then use the Hamiltonian

$$\gamma(\tau, \vec{x})\, H(\vec{x}, \vec{p}, \tau).$$

However, quantumly, this product is ambiguous.
↳ the product is not self-adjoint, and there are many ways to produce a self-adjoint operator. → this will not matter if $\langle \hat{H}_{NR} \rangle \ll mc^2$.

One possibility is to consider the Hamiltonian

$$\hat{\mathcal{H}}(\hat{\vec{x}}, \hat{\vec{p}}, \tau) = \frac{1}{2}\left(\Upsilon(\tau, \hat{\vec{x}})\, H(\hat{\vec{x}}, \hat{\vec{p}}, \tau) + \text{H.c.}\right)$$

The dynamics are given by

$$i\frac{\partial |\psi\rangle}{\partial \tau} = \hat{\mathcal{H}}(\hat{\vec{x}}, \hat{\vec{p}}, \tau)\, |\psi\rangle.$$

In F.N.C. $\quad \gamma(\tau, \vec{x}) = 1 + a_i x^i + \frac{1}{2} R_{0i0j} x^i x^j + \mathcal{O}(|x|^3, \tau^2, a^2)$

so $\quad \hat{\gamma}(\tau)\cdot \hat{H}(\tau) = \left(1 + a_i \hat{x}^i + \frac{1}{2} R_{0i0j} \hat{x}^i \hat{x}^j\right)(m + \hat{H}_{NR})$

$$= m\gamma(\tau) + \hat{H}_{NR} + \underline{\text{small terms}}$$

↳ on the size
↳ on the curvature
↳ on the acc.
↳ on the non-rel. energy.

$$= \underbrace{m + \hat{H}_{NR}(\tau)}_{\hat{H}(\tau)} + m a_i \hat{x}^i + \frac{m}{2} R_{0i0j} \hat{x}^i \hat{x}^j + \dots$$

⇒ The correction to the Hamiltonian is approximately $\; + m a_i \hat{x}^i + \frac{m}{2} R_{0i0j} \hat{x}^i \hat{x}^j$

↳ quadrupole coupling to gravity.

↳ dipole coupling to acceleration.

refer again to arXiv:2206.01225

Breakdown of covariance

Microcausality condition: $[\hat{O}(x), \hat{O}(x')] = 0$ for $x \asymp x'$ (spacelike)

↳ this condition is essential so that the time ordering is uniquely defined (covariant)

$$T(\hat{O}(x)\hat{O}(x')) = \Theta(t-t')\,\hat{O}(x)\hat{O}(x') + \Theta(t'-t)\,\hat{O}(x')\hat{O}(x)$$

$$= \Theta(t-t')\,\hat{O}(x)\hat{O}(x') + \Theta(t'-t)\left(\hat{O}(x)\hat{O}(x') + [\hat{O}(x'), \hat{O}(x)]\right)$$

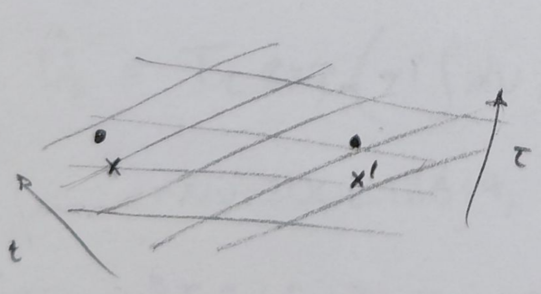
$$= \hat{O}(x)\hat{O}(x') + \Theta(t'-t)\,[\hat{O}(x'), \hat{O}(x)]$$

$$\stackrel{!}{=} \hat{O}(x)\hat{O}(x') + \Theta(\tau'-\tau)\,[\hat{O}(x'), \hat{O}(x)]$$

$\Leftrightarrow \left(\Theta(t'-t) - \Theta(\tau'-\tau)\right)[\hat{O}(x'), \hat{O}(x)] = 0 \quad \forall\, t, \tau$ time coordinates

Picking arbitrary time coordinates, the first term in the product can always be made non-zero for spacelike separated x and x'.

e.g.



THE TIME ORDERING IS IMPORTANT:

$$\hat{U} = \underline{\underline{T}}\exp\left(-i\int dV\,\hat{h}_I(x)\right)$$

this is not $T\left(\exp\left(-i\int dV\,\hat{h}_I(x)\right)\right)$!

Now, in the smeared UDW model:

$$\hat{h}_I(x) = \lambda \Lambda(x) \hat{\mu}(\tau) \hat{\phi}(x)$$

$$\Rightarrow [\hat{h}_I(x), \hat{h}_I(x')] = \lambda^2 \Lambda(x)\Lambda(x') \hat{\phi}(x)\hat{\phi}(x') [\hat{\mu}(\tau), \hat{\mu}(\tau')] \text{ for } x \,⫫\, x'.$$

but $\hat{\mu}(\tau) = \sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}$

$$\Rightarrow \hat{\mu}(\tau)\hat{\mu}(\tau') = \sigma^+\sigma^- e^{i\Omega(\tau-\tau')} + \sigma^-\sigma^+ e^{-i\Omega(\tau-\tau')}$$

$$\Rightarrow [\hat{\mu}(\tau), \hat{\mu}(\tau')] = \underbrace{\left(e^{i\Omega(\tau-\tau')} - e^{-i\Omega(\tau-\tau')}\right)}_{2i\sin(\Omega(\tau-\tau'))}\underbrace{(\sigma^+\sigma^- - \sigma^-\sigma^+)}_{\hat{\sigma}_z}$$

$$\Rightarrow [\hat{h}_I(x), \hat{h}_I(x')] = 0 \iff \tau - \tau' = \frac{2n\pi}{\Omega} \text{ (almost never)}$$

$\Rightarrow \hat{U}_I$ depends on a notion of time ordering.

It is a non-relativistic model, and τ is naturally priviledged. So we have:

$$\hat{U}_I = T_\tau \exp\left(-i\int dV\, \hat{h}_I(x)\right)$$

Also, in arXiv:2006.12514, it was shown that

$$\hat{\rho}_D^t - \hat{\rho}_D^\tau = \lambda^2 [\hat{\sigma}_z, \hat{\rho}_{D,0}] \cdot E + \mathcal{O}(\lambda^4),$$

$$E = -2i \int_{S_\leq} dV dV' \Lambda(x)\Lambda(x') W(x,x') \cdot \sin(\Omega(\tau-\tau'))\Theta(t'-t)$$

$\hookrightarrow S_\leq = \{(x,x') \in M \times M' : x \,⫫\, x' \;\&\; \tau > \tau' \Rightarrow t' \leq t\}$

- there is no leading order discrepancy if
$\hat{\rho}_{D,0} = \rho_g |g\rangle\langle g| + \rho_e |e\rangle\langle e|$.

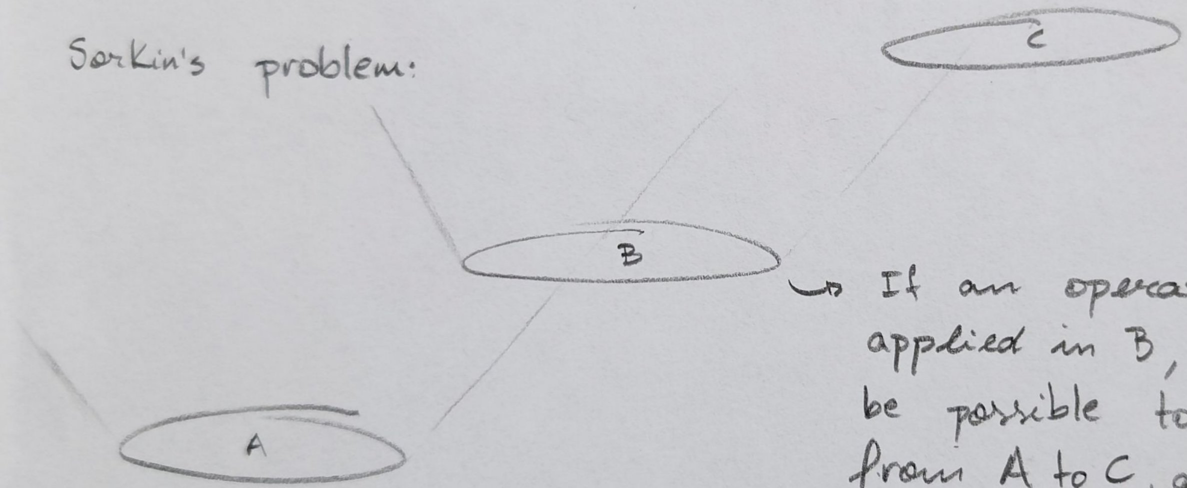
- gapless case is safe. (no time order)

- the difference depends on the region where the times differ (spacelike) integrated over the support of $\Lambda(x)\Lambda(x')$.

$\Rightarrow$ point-like detectors are fine

<u>Causality</u> <u>Violations</u>

Sorkin's problem:



$\rightarrow$ If an operation* is applied in B, it might be possible to signal from A to C, although $A \not\sim C$.

- Of course, particle detectors allow for this type of issue $\rightarrow$ the reason is again one quantum d.o.f. coupled to multiple spacelike separated points. $\Rightarrow$ point-like detectors are OK.

arXiv: 2102.03408 } detectors
arXiv: 2305.07756 }
arXiv: gr-qc/9302019 $\rightarrow$ Sorkin's Original
arXiv: 1912.06141 $\rightarrow$ modern formulation
arXiv: 2106.09027 $\rightarrow$ even in QFT one has to be carefull.